Quiz 2A, Calculus I - No calculators

Dr. Graham-Squire, Fall 2017

Name: Key

1. (3 points) Prove the derivative rule that $\frac{d}{dx}(c \cdot g(x)) = c \cdot g'(x)$, where c is any constant, by one of the following methods:

   - Using the definition of the derivative $\left(f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}\right)$, or
   - Using the product rule and the rule for the derivative of a constant.

$$\frac{d}{dx}(c \cdot g(x))$$

$$= \frac{d}{dx}(c) \cdot g(x) + \frac{d}{dx}(g(x)) \cdot c$$

$$= \cancel{0 \cdot g(x)} + g'(x) \cdot c$$

$$= c \cdot g'(x)$$

$$f(x) = c \cdot g(x)$$

$$f(x+h) = c \cdot g(x+h)$$

$$\frac{d}{dx}(c \cdot g(x))$$

$$= \lim_{h\to 0} \frac{c \cdot g(x+h) - c \cdot g(x)}{h}$$

$$= c \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= c \cdot g'(x)$$

2. (4 points) Calculate the derivative of $h(x)$. You do not need to simplify.

$$h(x) = \frac{(\tan x)e^x}{7x^5 - \pi x + 17}$$

$$h'(x) = \frac{(7x^5 - \pi x + 17)(\sec^2 x \cdot e^x + \tan x \cdot e^x) - (\tan x)e^x \cdot (35x^4 - \pi)}{(7x^5 - \pi x + 17)^2}$$

3. (3 points) Calculate the derivative of $m(x)$, and simplify your answer (hint: it may help to simplify *before* you take the derivative).

1.5 for deriv.
1.5 for simp.

$$m(x) = \frac{2x^6 - 3x^5 + x^4}{x^5}$$

$m(x) = (2x^6 - 3x^5 + x^4)x^{-5}$

$m(x) = 2x - 3 + x^{-1}$

$m'(x) = 2 - 0 - 1x^{-2}$

$\boxed{m'(x) = 2 - x^{-2}}$

or $2 - \frac{1}{x^2}$

or

$$m'(x) = \frac{x^5(12x^5 - 15x^4 + 4x^3) - 5x^4(2x^6 - 3x^5 + x^4)}{(x^5)^2}$$

$$m'(x) = \frac{12x^{10} - \cancel{15x^9} + 4x^8 - 10x^{10} + \cancel{15x^9} - 5x^8}{x^{10}}$$

$$= \frac{2\cancel{x^{10}}}{\cancel{x^{10}}} - \frac{x^8}{x^{10}}$$

$$= \boxed{2 - \frac{1}{x^2}} \quad \text{or} \quad \frac{2x^2 - 1}{x^2}$$

Quiz 2B, Calculus I - No calculators

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2.5 min $\Rightarrow$ 15 min.

Name: Key

1. (3 points) Calculate the derivative of $m(x)$, and simplify your answer (hint: it may help to simplify *before* you take the derivative).

$$m(x) = \frac{3x^7 - 8x^6 + x^5}{x^6}$$

1.5 for deriv

1.5 for simp.

$$m(x) = \frac{3x^7}{x^6} - \frac{8x^6}{x^6} + \frac{x^5}{x^6}$$

$$m(x) = 3x - 8 + x^{-1}$$

$$m'(x) = 3 - 0 - 1x^{-2}$$

$$= \boxed{3 - x^{-2}} \text{ or } \boxed{3 - \frac{1}{x^2}}$$

or

$$m'(x) = \frac{x^6(21x^6 - 48x^5 + 5x^4) - (3x^7 - 8x^6 + x^5)\cdot 6x^5}{(x^6)^2}$$

$$= \frac{21x^{12} - 48x^{11} + 5x^{10} - 18x^{12} + 48x^{11} - 6x^{10}}{x^{12}}$$

$$= \frac{3x^{12} - x^{10}}{x^{12}} = \frac{3x^{12}}{x^{12}} - \frac{x^{10}}{x^{12}} = \boxed{3 - \frac{1}{x^2}}$$

$$\text{or } = \frac{x^{10}(3x^2 - 1)}{x^{12}} = \boxed{\frac{3x^2 - 1}{x^2}}$$

2. (3 points) Prove the derivative rule that $\frac{d}{dx}(c \cdot g(x)) = c \cdot g'(x)$, where c is any constant, by one of the following methods:

- Using the definition of the derivative $\left(f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right)$, or
- Using the product rule and the rule for the derivative of a constant.

$$\frac{d}{dx}(c \cdot g(x))$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c \cdot (g(x+h)) - c \cdot g(x)}{h}$$

$$= c \left(\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \right)$$

$$= c \cdot g'(x)$$

or

$$\frac{d}{dx}(c \cdot g(x))$$

$$= \frac{d}{dx}(c) \cdot g(x) + \frac{d}{dx}(g(x)) \cdot c$$

$$= \cancel{0 \cdot g(x)} + g'(x) \cdot c$$

$$= c \cdot g'(x)$$

3. (4 points) Calculate the derivative of $h(x)$. You do not need to simplify.

$$h(x) = \frac{(3x^8 - \pi x)(\tan x)}{e^x + 6}$$

$$h'(x) = \boxed{\frac{(e^x+6)\left((24x^7 - \pi)(\tan x) + \sec^2 x\,(3x^8 - \pi x)\right) - (3x^8 - \pi x)(\tan x)(e^x)}{(e^x+6)^2}}$$

−1 if no product rule